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Propagation of a blast wave in uniform or non-uniform media: a uniformly valid analytic solution

By P. L. SACHDEV
Department of Physics, University of Toronto

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The shock propagation theory of Brinkley & Kirkwood (1947) is extended to provide a uniformly valid analytic solution of point-explosion problems both when the undisturbed medium is uniform and when it is stratified. This is achieved mainly by selecting the parameter expressing a similarity restraint in this theory such that initially it gives precisely the Taylor–Sedov solution, while asymptotically, in the weak regime, still retaining the well-known Landau–Whitham–Sedov form of the solution for shock overpressure. The shock overpressure, as calculated by the present method for spherical and cylindrical blast waves in the entire regime from the point of explosion to where they have become very weak, shows excellent agreement with that from the exact numerical solutions of Lutzky & Lehto (1968) and Plooster (1970). The solution for a spherical shock propagating in an exponential atmosphere stratified by a constant acceleration due to gravity also shows a good agreement with the exact numerical solution of Lutzky & Lehto.

1. Introduction

The propagation of a blast wave has been studied by a number of investigators, particularly since World War II. The governing nonlinear partial differential equations with the boundary conditions on the unknown shock surface (the Rankine–Hugoniot conditions) pose such serious mathematical difficulties that, even with the simplified physical assumption that the entire energy of the blast is released at a point, a uniformly valid analytic solution has not been found. The well-known Taylor–Sedov solution for a point explosion, based on similarity and dimensional considerations, shows good agreement with the experimentally measured shock trajectory only up to a few tens of metres. Sakurai (1953, 1954, 1965) attempted to improve upon this solution by a perturbation technique with the inverse square of the shock Mach number as the perturbation parameter. The solution was thus rendered more accurate for greater distances from the point of explosion but soon began to depart from the exact numerical solution. On the other hand, Landau (1945), Whitham (1950) and Sedov (1959), with different approaches, obtained an asymptotic form of the solution in the weak shock regime when the blast has propagated far from the source. It is the intermediate shock-strength regime which has eluded an analytic treatment. For the inhomogeneous medium a few similarity solutions for the blast wave (see Sedov 1959) have been obtained but these are special solutions, valid only under restrictive similarity conditions. Laumbach & Probstein (1969) have recently given an analytic approach for strong shock propagation in an exponential medium. This approach is based on the assumption that almost the entire mass of the blast is concentrated on the shock surface, an assumption similar to that of Chernyi (1959), who employed a perturbation technique to study shock propagation.

When looking for an analytic approach which would be uniformly valid over the entire course of the blast propagation it was realized that the shock propagation theory of Brinkley & Kirkwood (1947) provided an appropriate framework. This theory had, however, to be suitably modified to meet the above requirement – a task similar to that of Hayes (1968), who modified the well-known Chisnell, Chester & Whitham method for shock propagation in the light of his exact numerical calculations. This method, however, is suitable only for taking into account the local non-uniformities which the shock encounters as it propagates, and is not capable of treating the blast-wave problem.

The Brinkley–Kirkwood (BK) theory for one-dimensional spherical, cylindrical and plane symmetric cases may be summarized as follows. The hydrodynamic equations of motion and continuity are specialized at the shock front and the Rankine–Hugoniot equation, expressing the conservation of momentum across the shock, is differentiated such that the shock is stationary to provide three equations for the four partial derivatives of pressure and velocity with respect to time and distance at the shock front. A fourth approximate equation is obtained by imposing a similarity constraint on the energy–time curve. These equations are solved simultaneously to obtain the variation of shock overpressure with distance. Also, the rate of decay of shock energy is obtained by the following consideration. As a particle crosses a shock its entropy and internal energy increase. This particle, with the new value of entropy, expands adiabatically until it comes to its original pressure (but higher temperature) and then it radiates energy at constant pressure, finally assuming its ambient value of pressure and specific volume. (This modified argument is due to Schatzman (1949).) Taylor (1950) has also used the same p, v path to evaluate the part of explosion energy which is degraded as heat and is thus not available for doing work as the shock propagates. This process is expressed mathematically as an equation describing the dissipation of the energy of explosion as heat. Besides this idealization of the particle path in the p, v plane, the similarity restraint mentioned above refers to the observation that the parameter

$$\nu(R) = \int_0^\infty f(R,\tau)\,d\tau, \tag{1.1}$$

where $f(R,\tau) = r^\alpha p'u'/R^\alpha pu$ is the energy–time integrand normalized by its peak value $R^\alpha pu$ at the shock front, and expressed as a function of distance R and a reduced time τ which normalizes its initial slope to -1, is a very slowly varying function of R. Therefore, it may be taken to be constant and obtained from the experiments. (This parameter equals unity if $f(R,\tau) = e^{-\tau}$.) This approximation is further justified by arguing that it is equivalent to the principle underlying the Rayleigh–Ritz method.

In the present paper we derive the equations for the variation of shock energy and pressure, following the above arguments both when the ambient medium is uniform and when it is non-uniform. Specifically, for the latter we consider an isothermal atmosphere with an exponential variation of pressure and density and a constant acceleration due to gravity (see Lutzky & Lehto 1968). Now, in the equation expressing energy dissipation, if the shock strength is allowed to tend to infinity the rate of shock energy decrease, dD/dR, tends to zero. The second equation, with D = constant, leads to the solution which has the same form as the Taylor–Sedov solution. We now choose the similarity parameter ν such that our solution exactly coincides with this solution. Thus, ν becomes a parameter dependent on $\gamma = c_p/c_v$, the ratio of specific heats, and on α ($\alpha = 2,1$ for the spherical and cylindrical symmetric cases respectively). If in the weak shock limit we choose $\nu = \frac{2}{3}$, the solution for the homogeneous medium again coincides with the well-known asymptotic formulae (Whitham 1950; Sedov 1959). We take the value of ν to be the one given by Taylor–Sedov solution until the (non-dimensional) shock overpressure $\Delta p = (p-p_0)/p_0 \lesssim 0{\cdot}02$, after which ν is decreased gradually to $\frac{2}{3}$ when $\Delta p \to 0$. However, the solution at this stage becomes insensitive to the value of ν since the term containing ν in the differential equation for pressure becomes very small in comparison with the other term. The important assumption that is made here is that this value of ν works for the inhomogeneous case too since its approximation is based on the thermodynamic behaviour of the particle after it crosses the shock, and is therefore independent of the inhomogeneity of the medium (cf. Dumond *et al.* 1946). The numerical results seem to justify this assumption.

The outline of this paper is as follows. Section 2 gives the general differential equations governing pressure and shock energy variation as the shock propagates. Section 3 deals with the uniform ambient atmosphere, while the inhomogeneous ambient medium is discussed in § 4. The shape of a strong shock as it propagates is considered in § 5 and concluding remarks are given in § 6.

2. Equations for shock overpressure and energy

The details of the derivation of the equations governing the variation of shock overpressure and energy as the shock propagates are omitted here. The papers by Brinkley & Kirkwood (1947) and Nadezhin & Frank-Kamenetskii (1965) may be referred to for this purpose. The function D occurring therein is identified as $E_0/2^\alpha\pi$ initially, where E_0 is the energy of explosion at the source per unit area/length for $\alpha = 2,1$, for spherical and cylindrical symmetry respectively. Subsequently it is the work function, giving the available mechanical energy in the shock at a particular time. Thus, after introducing the factor $2^\alpha\pi$ in D, we have

$$\frac{dp}{dR} = \frac{1}{L}\left[\frac{-(2R)^\alpha \pi p^3 G\nu(\alpha,\gamma)}{P_0 D}\frac{1-y}{z-1} - \frac{\alpha p}{R}(2y+(1-y)G) + 2p\left\{\left(\frac{1}{\rho_0}+\frac{1}{U}\frac{\partial U}{\partial \rho_0}\right)\frac{d\rho_0}{dR} + \frac{1}{U}\frac{\partial U}{\partial P_0}\frac{dP_0}{dR}\right\}\right], \tag{2.1}$$

$$dD/dR = -(2R)^\alpha \pi \rho_0 h(p), \tag{2.2}$$

where $$h(p) = \frac{1}{2}\frac{P_0}{\rho_0}\left(\frac{P}{P_0}-1\right)\left(\frac{\rho_0}{\rho}+1\right)+\frac{P_0}{\rho_0}\left[\frac{\gamma}{\gamma-1}\frac{P}{P_0}\left(\frac{\rho_0}{\rho}\right)\left\{\left(\frac{P}{P_0}\right)^{(1-\gamma)/\gamma}-1\right\}\right]$$

$$= \frac{\gamma P_0}{\rho_0}\frac{z^2-1-z\left(z+\dfrac{\gamma+1}{\gamma-1}\right)(1-z^{-(\gamma-1)/\gamma})}{\gamma-1+(\gamma+1)z}. \tag{2.3}$$

Here $p = P-P_0$ is the pressure at the shock in excess of the ambient pressure just ahead of it, at a distance R from the source; $y = \rho_0/\rho$, $z = P/P_0$ are measures of shock strength expressed by ratios of density and pressure across the shock respectively. The shock velocity U is a function of the overpressure p; the undisturbed pressure is $P_0(R)$ and the density $\rho_0(R)$. In the derivation of the above equations we have assumed the presence of a constant gravitational acceleration g with a view to solving the problem of shock propagation in an exponential atmosphere (see Lutzky & Lehto 1968) in § 4. The parameter ν is a function of the symmetry exponent α and γ ($=c_p/c_v$). The functions G and L are defined as

$$G = 1-\left(\frac{\rho_0 U}{\rho c}\right)^2, \tag{2.4}$$

$$L = 2+2\left(1-\frac{p}{U}\frac{\partial U}{\partial p}\right)-G, \tag{2.5}$$

where c is the speed of sound behind the shock. It may be noted that the equation of conservation of entropy along the particle line behind the shock is not explicitly used. Instead, the particle path in the p, v plane is followed from physical considerations; this path implies shedding of mechanical energy by the shock as heat, and hence its decay.

3. Uniform ambient medium

First we consider the case of a blast wave propagating into a uniform medium such that P_0 and ρ_0 are constant. In the limiting case of infinite shock strength, $h(p)\to 0$ so that the energy of the blast is conserved as mechanical energy available for external work, and the solution of (2.1) reduces to the Taylor–Sedov solution as was shown in an earlier paper (Sachdev 1971, hereinafter referred to as I) provided that $\nu(\alpha,\gamma)$ is chosen suitably from the exact numerical solution of Taylor (1950) and Sakurai (1953). Table 1 gives these values for $\alpha = 2$, 1 for different values of γ.

In the limit $z\to 1$ it can be easily verified that the (2.1) and (2.2) reduce to

$$\frac{dp}{dR}+\frac{\alpha p}{2R} = -\frac{\gamma+1}{12\gamma^2}(2R)^\alpha\frac{\pi p^4}{DP_0^2}, \tag{3.1}$$

$$\frac{dD}{dR} = -\frac{\gamma+1}{12\gamma^2}(2R)^\alpha\frac{\pi p^3}{P_0^2}, \tag{3.2}$$

whose solution gives the correct asymptotic form for shock overpressure in this limit, namely $Rp = P_1(\log R/R_1)^{-\frac{1}{2}}$ for the spherically symmetric case and $R^{\frac{1}{2}}p = P_1[2(R^{\frac{1}{2}}-R_1^{\frac{1}{2}})]^{-\frac{1}{2}}$ for the cylindrically symmetric case provided ν is chosen to be $\frac{2}{3}$. Thus the solution for the blast wave problem as obtained by the BK theory has been forced to coincide with the exact solution in the limit of very strong and very weak shocks. Furthermore, if the value of ν is taken to be the one given by the Taylor–Sedov solution until $p \lesssim 0{\cdot}02$ the numerical results obtained from (2.1) and (2.2) show extremely good agreement with the exact numerical solution (see figures 1 and 2). In fact, in the weak shock regime, $p \lesssim 0{\cdot}02$, the contribution of the first term in (2.1), which involves ν and D, is very much smaller

α/γ	1·6667	1·4	1·3	1·2
2	2·3807	3·1158	3·7388	4·8097
1	2·574	3·4078	—	5·363

TABLE 1. Similarity parameter ν as a function of α and γ

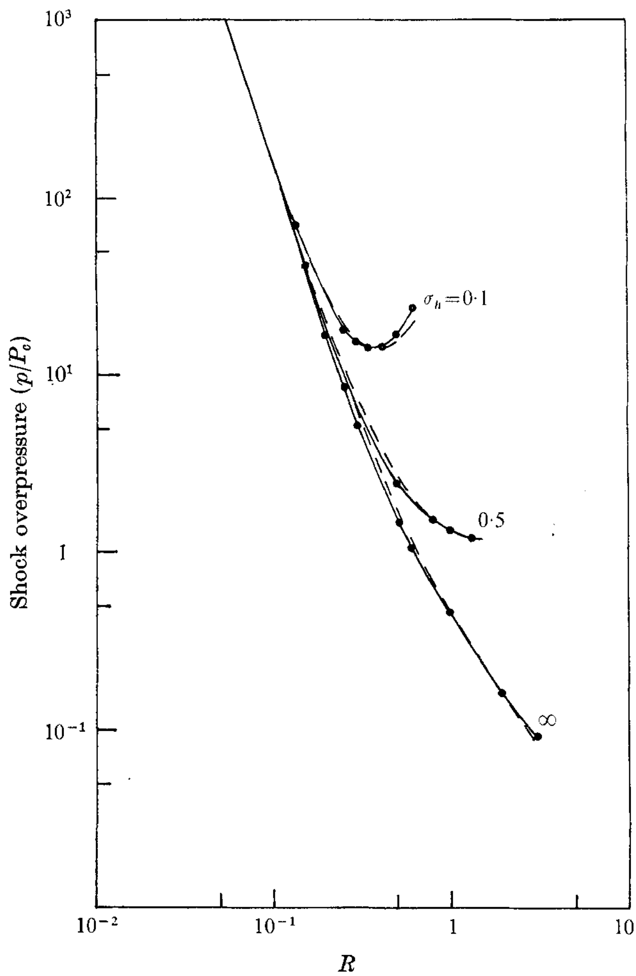

FIGURE 1. Shock overpressure *vs.* radius for various values of $\sigma_h = h(P_0/E_0)^{\frac{1}{3}}$ for a spherically symmetric blast. ---, exact numerical solution, Lutzky & Lehto (1968); ——, present theory.

than the second term, so that change in the value of ν will not make a significant difference to the solution. However, to conform with the analytic solution of (3.1) and (3.2) ν may be slowly altered from its value ν_0 to $\frac{2}{3}$ over the infinitely long range in which the weak shock overpressure tends to zero. Figure 1 ($\sigma_h = \infty$) gives the variation of shock overpressure with radius for a spherical blast wave propagating in air ($\gamma = 1{\cdot}4$), while figure 2 shows the corresponding solution for a cylindrical blast wave. The results obtained by the present theory agree very well with the exact numerical solutions of these problems by Lutzky & Lehto (1968) and Plooster (1970).

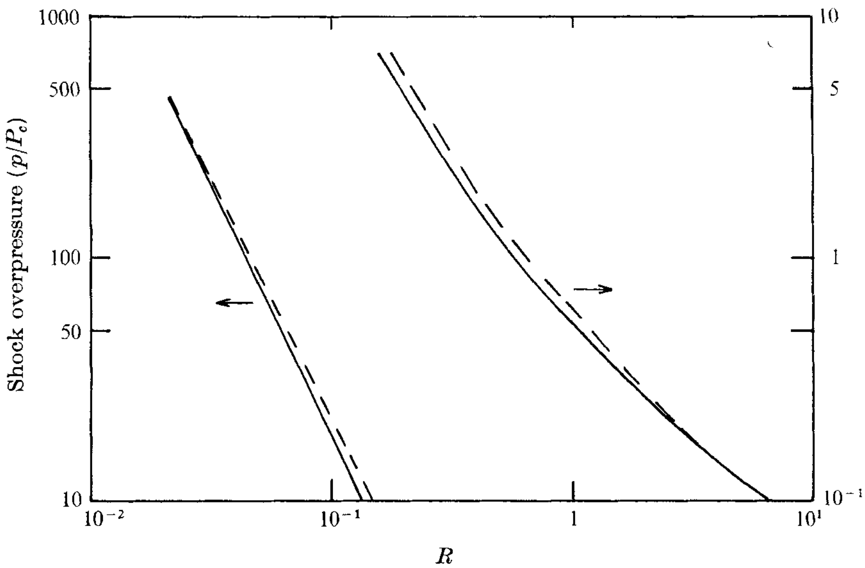

FIGURE 2. Shock overpressure *vs.* radius for a cylindrically symmetric blast. ---, exact numerical solution, Plooster (1970); ——, present theory.

4. Non-uniform medium

In this section we consider the propagation of a blast wave in an exponential atmosphere. This problem has been numerically studied by Lutzky & Lehto (1968). At time $t = 0$ an amount of energy E_0 is released at the origin of a spherically symmetric isothermal atmosphere. The initial stratification of pressure and density is determined by the condition that the atmosphere be in equilibrium with a constant gravitational acceleration $\bar{g}$, acting radially outward. Thus the initial pressure and density distributions have the form

$$P_0 = P_c \exp(r/h), \tag{4.1}$$

$$\rho_0 = \rho_c \exp(r/h). \tag{4.2}$$

Here r is the distance measured from the origin of explosion, P_c and ρ_c are the pressure and density at the origin respectively, $h = P_c/\rho_c \bar{g}$ is the scale height of the atmosphere and $\bar{g} = g\cos\phi$, the component of acceleration due to gravity in a direction making an angle ϕ with the downward vertical. While the atmosphere is assumed to be spherically symmetric, the density and pressure variation along a ray is taken according to (4.1) and (4.2). To keep the calculations stable in the spherically symmmetric system a uniform gravitational acceleration is included in the equations. For this particular model (2.2) and (2.3) are simplified with the help of the Rankine–Hugoniot conditions and (4.1) and (4.2). The over-pressure p, density ρ, distance R and the energy function D are normalized by their central undisturbed values P_c, ρ_c, $(P_c/E_0)^{\frac{1}{3}}$ and E_0 respectively. Finally, we have the equations

$$\frac{dp}{dR} = \frac{1}{L}\left\{\frac{-4\pi\nu(\gamma,2)\,R^2p^2G(1-y)}{D} - \frac{p}{R}(4y+2(1-y)\,G) + \frac{p}{\sigma_h}\left(1+\frac{\lambda^2+1}{z+\lambda^2}\right)\right\}, \quad (4.3)$$

$$\frac{dD}{dR} = -4\pi\gamma R^2 \exp\left(\frac{R}{\sigma_h}\right).\frac{z^2-1-z[z+(\gamma+1)/(\gamma-1)]\,(1-z^{-(\gamma-1)/\gamma})}{\gamma-1+(\gamma+1)z}, \quad (4.4)$$

$$z = 1+p, \quad y = \frac{1+\lambda^2 z}{z+\lambda^2}, \quad G = \frac{(\gamma+1)}{2\gamma}\frac{(z-1)}{z}, \quad \lambda^2 = \frac{\gamma-1}{\gamma+1},$$

$$L = \frac{5\gamma-1}{2\gamma} + \frac{\gamma+1}{2\gamma}\frac{1}{z} + \frac{1+\lambda^2}{z+\lambda^2}.$$

Here $\nu(\gamma, 2)$ is the similarity parameter for the spherically symmetric case obtained from the Taylor–Sedov solution as explained in § 3. It is implicitly assumed that this parameter serves for the inhomogeneous medium also. The non-dimensional parameter

$$\sigma_h = h(P_c/E_0)^{\frac{1}{3}} = \frac{1}{\rho_c g\cos\phi}\left(\frac{P_c^4}{E_0}\right)^{\frac{1}{3}}$$

is a measure of the importance of the inhomogeneity, initial energy of explosion and the density and pressure at the origin in the undisturbed atmosphere. It also describes a whole range of problems for different combinations of these constants. For example, if $g \to 0$ so that $\sigma_h \to \infty$, we have a uniform undisturbed atmosphere with constant pressure P_c and density ρ_c. As the value of σ_h decreases, the atmospheric inhomogeneity strengthens if other parameters (P_c and E_0) are held constant. Similarly, for constant values of the scale height h (which is a measure of stratification) and the central pressure P_c, a smaller value of σ_h corresponds to larger value of the energy of explosion E_0.

To compare our results with those from the exact numerical solution of Lutzky & Lehto (1968) we integrated (4.1) and (4.2) with the initial conditions at $R_0 = 0{\cdot}025$, $D = 1$ and $p = 10\,005$ as obtained from the von Neuman formula $p = 0{\cdot}157/R^3$, assuming the solution up to $R_0 = 0{\cdot}025$ to be governed by the Taylor–Sedov–von Neuman solution. It is well known that in the initial stages ($R \sim 0{\cdot}1$) the effect of non-homogeneities on blast-wave propagation is negligible. This is borne out by the present calculations as well as by those of Lutzky & Lehto. Figure 1 shows the comparative study of results obtained by the present theory and the exact numerical solution of Lutzky & Lehto for $\sigma_h = \infty$, 0·5 and 0·1. The agreement is excellent even for $\sigma_h = 0{\cdot}1$, when the inhomogeneities of the undisturbed medium become very strong. It is interesting to compare the magnitude of the three terms on the right-hand side of (4.1) which, respectively, give essentially the effect of the energy of explosion, the curvature of the shock and the atmospheric stratification. First, in the case of a uniform atmosphere, $\sigma_h = \infty$, the last term disappears. The other two terms play a comparable role in the initial stages of blast-wave propagation, the first being larger in magnitude than the second, but after the shock has propagated one dynamic length, $R \sim 1$, the second term assumes a major role. It becomes more important as the shock propagates further until finally the first term becomes negligible compared with the second when the shock has propagated far away from the source. When $\sigma_h \neq 0$ the last term, representing the influence of inhomogeneity of the medium, plays a role which depends on the strength of the inhomogeneity. For example when $\sigma_h = 0{\cdot}1$, so that the stratification is quite intense, the last term is more important than the other two except in the very early stages of shock propagation, when all the three are comparable.

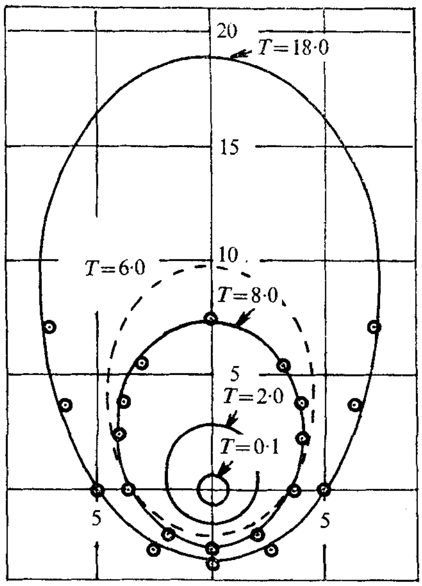

FIGURE 3. Shock envelope at different times for $\gamma = 1{\cdot}2$, with length scale Δ and $T = t(E/4\pi\rho_B\Delta^5)^{\frac{1}{2}}$. ——, Laumbach & Probstein; ----, Andriaken *et al.*; $\odot$, present theory.

5. Shape of strong shock

We briefly discuss the shape of a strong blast wave as it propagates, under the assumption of local radiality (Laumbach & Probstein 1969; Lutzky & Lehto 1968). Figure 3 shows the comparison of the shock shape as obtained by Laumbach & Probstein (1969), Andriakin *et al.* (1962) and the present theory. The formulae in our case are easily obtained from I:

$$\dot{R}^2 = \frac{\gamma(\gamma+1)}{2\nu(\gamma)}\left(\frac{5\gamma-1}{2\gamma}\right)^{-2(\gamma^2+6\gamma-3)/(\gamma+1)(5\gamma-1)} \exp \pm \left[\left(\frac{3\gamma-1}{5\gamma-1}\right) R\cos\theta\right]$$
$$\times R^{-4(2\gamma^2-\gamma+1)/(\gamma+1)(5\gamma-1)}\left[\int_0^{2\gamma R/(5\gamma-1)} z^{2(\gamma^2+6\gamma-3)/(\gamma+1)(5\gamma-1)}\, e^{\mp z\cos\theta}\, dz\right]^{-1}, \quad (5.1)$$

$$t = \int_0^R \frac{dR}{\dot{R}}. \tag{5.2}$$

In the above, the upper and lower sign correspond to the medium with exponentially decreasing and increasing density respectively, and θ gives a particular ray direction. It is found that the agreement of all the three approaches is excellent in the lower half of the shock surface which propagates in a medium with exponentially increasing density. In the upward direction, the results corresponding to the present theory lie close to those of Laumbach & Probstein (1969) except for the upper portion of the shock after the shock has propagated far away (figure 3). In fact, after propagating a certain distance the strong shock assumption becomes invalid. This, at least partly, explains the disagreement for the results as obtained by the present theory with those of Laumbach *et al.* shown in I. The shock velocity climbs much faster after the minimum in our case and therefore the shock goes to infinity earlier than in the case of Laumbach & Probstein (1969).

6. Concluding remarks

A modification of the Binkley–Kirkwood theory is presented here and gives an excellent description of the propagation of a blast wave originating from a point source in the entire regime from the stage when it is very strong right up to its final decay. The results for a spherically symmetric exponential atmosphere agree very well with the exact numerical solution of Lutzky & Lehto (1968), even when the inhomogeneity is strong. The important feature of the BK theory is its emphasis on the dissipation of the shock-wave energy as heat and hence the loss of mechanical energy for doing work against the undisturbed pressure, and on the subsequent shock decay. This point has also been noted by Taylor (1950), who referred to this heat loss as the energy wasted and unavailable for shock propagation. Since this theory takes account of this energy-shedding right from the beginning, there will be slight departure from the Taylor–Sedov solution in the early stages of blast propagation since the Taylor–Sedov solution coincides with the present solution only when the shock strength tends to infinity. The right-hand side of (2.2) tends to zero when $z \to \infty$, but as z becomes finite even though large, it begins to rise sharply and then decreases finally, to settle down to a small value when the shock has become comparatively weak. It is worth while to mention that the effect of changing the initial point, for example from $R_0 = 0{\cdot}05$ to $R_0 = 0{\cdot}025$, is not significant except in the very early stages of propagation, when the explosion energy begins to dissipate earlier. In any case the solution in the early stages, as mentioned above, will differ slightly from the Taylor–Sedov solution, wherein the shock strength is infinitely large, while in the present theory the shock strength initially is very large but finite. On the other hand, it is remarkable that the minimum for the shock strength in the inhomogeneous case remains almost unaltered by a small shift in the initial point.

It is not so simple to find the details of flow behind the shock in the present case as was the case in I. The formulae are comparatively cumbersome and are therefore omitted here.

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